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HYDRODYNAMIC STABILITY OF A CYLINDRICAL REACTION FRONT ASSOCIATED WITH A STRONG INCREASE OF VISCOSITY

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Hydrodynamic stability of a plane chemical reaction front in a gas was first considered in [1] neglecting transport effects. The effect of transport processes on the stability of a plane front in viscous gases and in condensed phases was studied in [2-5]. The stability of a curved front was considered in [6] for the example of the propagation of a spherical flame in a gas at rest. Transport effects were assumed to be small in this case and were taken into account phenomenologically in the Markstein approximation [7].

In the present paper we consider, in the linear approximation, the hydrodynamic stability of a stationary cylindrical reaction front in a radial, axisymmetric flow of a condensed medium. The flow is generated with the help of two coaxial, permeable, cylindrical surfaces. It is assumed that the viscosity of the medium is significantly increased by the reaction process (this is typical of polymerization reactions, for example) and hence inertial effects are small and are not taken into account [8, 9]. We study the dependence of the perturbation increment of the stationary states of the front on the parameters of the problem: the ratio of the viscosities of the medium on the front, the ratios of the radii of the boundary surfaces to the radius of the front, and the resistances of the order approximation in the (small) ratio of the viscosities this dependence is obtained analytically. It is shown that the front is absolutely stable in nearly the entire physical region of the parameter space. The front becomes unstable only when it approaches the outer boundary surface and the surface has a small resistance.

We note that it was found in a number of papers (see [8-12], for example) that for channel flow of a reacting medium, whose viscosity increases in the process of the reaction, the reaction front is sharply curved and drawn out near the channel axis (the "rupture" phenomenon). Our study of the hydrodynamic stability of a cylindrical front shows that small distortions of the front are damped for a wide range of the parameters and hence a "rupture" in the radial direction does not occur.

1. We assume the thickness of the front to be small compared to the distance between the surfaces bounding the flow and therefore treat it as a surface of discontinuity propagating with a constant velocity U with respect to the reactions medium (the local Michelson law [13]). The density of the medium is taken to be constant.

With these assumptions, the motion of the medium in front of and behind the front is described by the equation of continuity and the Stokes equation

$$\nabla p_{1,2} = \mu_{1,2}\nabla^2\mathbf{V}_{1,2}; \tag{1.1}$$

$$\nabla\cdot\mathbf{V}_{1,2} = 0, \tag{1.2}$$

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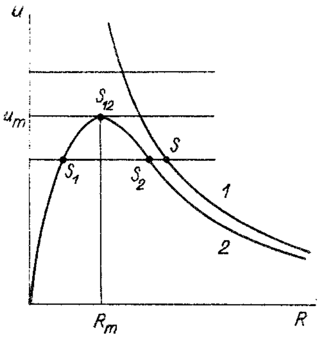

Fig. 1

where p is the pressure, μ is the viscosity, **V** is the velocity vector, and the subscripts 1 and 2 refer to the flow regions in front of and behind the front.

Boundary conditions to the system of equations (1.1) and (1.2) are imposed on the penetrable surfaces bounding the flow, and on the front. Since inertial effects are small, we assume that the normal component of the velocity on the penetrable surfaces is proportional to the pressure drop across them

$$(p_{10} - p_1)|_{R_1} = \sigma_1 V_{1n}|_{R_1}; \tag{1.3}$$

$$(p_2 - p_{20})|_{R_2} = \sigma_2 V_{2n}|_{R_2}. \tag{1.4}$$

Here $\sigma_{1,2}$ are proportionality constants (the resistances of the surfaces) and $R_{1,2}$ are the radii of the surfaces. The subscript 0 refers to the flow regions outside the penetrable surfaces. The tangential components of the velocity on the boundary surfaces are assumed to be equal to zero:

$$\mathbf{V}_{1\tau}|_{R_1} = 0; \tag{1.5}$$

$$\mathbf{V}_{2\tau}|_{R_2} = 0. \tag{1.6}$$

On the reaction front, in view of the "cohesion" of the flow and the incompressibility of the medium, the velocity vector is continuous

$$\mathbf{V}_1|_R = \mathbf{V}_2|_R, \tag{1.7}$$

where R is the surface of the front, and is given by the equation R(r, t) = 0, where r is the radius vector and t is the time.

Conservation of momentum on the surface of discontinuity [14], and the continuity of the velocity vector imply the continuity of the stress vector

$$\mathbf{P}_{n1}|_R = \mathbf{P}_{n2}|_R. \tag{1.8}$$

where $\mathbf{P}_n = \mathbf{P}\cdot\mathbf{n}$, **P** is the stress tensor, $\mathbf{n} = \nabla R/|\nabla R|$ is unit normal to the front in the direction of its propagation.

The condition that the propagation velocity of the front be constant with respect to the reacting medium is written in the form [14]

$$N - V_n|_R = U \tag{1.9}$$

($N = -(\partial R/\partial t)/|\nabla R|$ is the propagation velocity of the surface of the front along the direction of the normal).

Equations (1.1) and (1.2) with the boundary conditions (1.3)-(1.9) give a closed formulation of the problem. It is significant that the time derivative appears explicitly only in the boundary condition (1.9). This is because the problem is quasistationary, in view of the smallness of inertial effects: the front propagates in a given velocity field with a constant relative velocity U, and the velocity and pressure fields adjust themselves instantaneously to the changing position of the front.

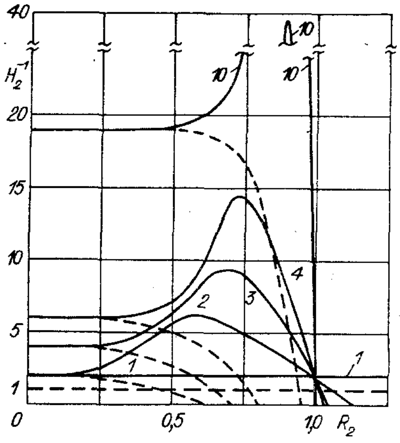

Fig. 2

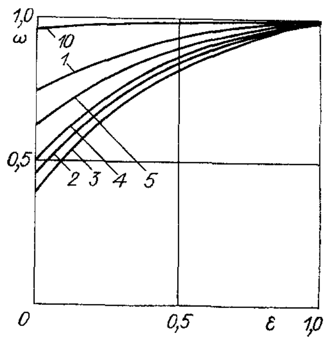

Fig. 3

2. We consider the stationary states of the front and their stability to one-dimensional perturbations which do not distort the cylindrical shape of the front. Introducing cylindrical coordinates (r, φ, z), where the z axis is along the flow axis, and putting $\partial/\partial\varphi = \partial/\partial z = 0$, $v_{1,2} = w_{1,2} = 0$, we find from (1.2) and (1.7)

$$u_1 = u_2 = q/2\pi r, \tag{2.1}$$

where u, v, w are the radial, angular, and axial components of the velocity vector **V**, respectively, and q is a constant of integration, which can be interpreted as an axisymmetric hydrodynamical source.

Substituting (2.1) into (1.1) and (1.8), we find that the pressure is constant in the flow regions in front of and behind the front, while across the front there is a pressure jump, due to the change in the viscosity

$$p_1 - p_2 = (\mu_2 - \mu_1)q/\pi R^2. \tag{2.2}$$

This relation can be interpreted as the result of an intrinsic hydraulic resistance of the front, which depends on the radius of the front [15].

The Michelson law (1.9) in this case has the form

$$dR/dt = u(R) - U \tag{2.3}$$

(u(R) is the local value of the flow velocity at the location of the front). The stationary states of the front are determined by the equation $u(R) = U$ $(dR/dt = 0)$. According to (2.1) in the case considered here flow velocity at a fixed point is uniquely related to the source and therefore the stationary states of the front and their stability to one-dimensional perturbations, which do not distort the shape of the front and flow, depend significantly on how the reacting medium is fed into the system [15].

If material is supplied at a constant rate q, then $u(R) = q/2\pi R$ is a monotonically decreasing function (Fig. 1, curve 1) and the front has a single stationary state (point S) determined by the radius

$$R^0 = q/2\pi U. \tag{2.4}$$

This stationary state is stable to one-dimensional perturbations. Indeed, putting $R = R^0 + R'$ into (2.3) (R' is a small perturbation) and linearizing the resulting equation with respect to R', we find

$$\frac{dR'}{dt} = \left.\frac{du(R)}{dR}\right|_{R^0} R', \tag{2.5}$$

and so $R' \sim \exp \omega t$, where

$$\omega = -q/2\pi(R^0)^2. \tag{2.6}$$

In the case of a source, $q > 0$ and therefore $\omega < 0$ and the front is stable (for a sink it is unstable, on the other hand).

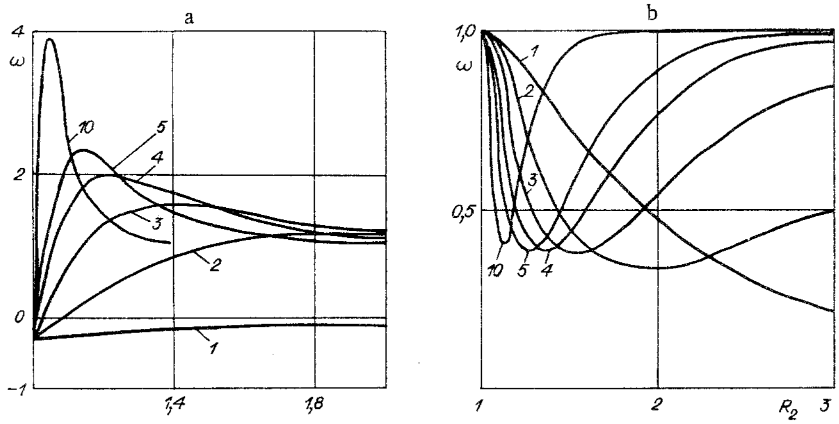

Fig.4

If material is supplied in a way such that the pressure drop $\Delta p = p_{10} - p_{20}$ between the permeable surfaces is constant, then, putting (2.1) into (1.3) and (1.4), and eliminating p_1, p_2, and q from the resulting equations and (2.2), we find

$$u(R) = \Delta p/(2\pi\sigma R + \Delta\mu/R),\ \sigma = \sigma_1/2\pi R_1 + \sigma_2/2\pi R_2,\\ \Delta\mu = \mu_2 - \mu_1. \tag{2.7}$$

This dependence (Fig. 1, curve 2) is nonmonotonic and has a maximum $u_m = \Delta p/4(\pi\sigma\Delta\mu)^{1/2}$ at $R = R_m = (\Delta\mu/\pi\sigma)^{1/2}$. The nonmonotonic nature of the dependence is due to the fact that on the one hand, u(R) should decrease with increasing R because of the increasing distance from the axis (see (2.1)), while on the other hand, q should increase because of the decrease in the hydraulic resistance of the front (see (2.2)).

If $U > u_m$ the front does not have stationary states: $dR/dt < 0$ and the front is pulled in toward the axis. If $U < u_m$ exist two stationary states (the points S_1 and S_2 in Fig. 1), which are determined by the radii

$$R^0_{1,2} = (\Delta p \mp \sqrt{\Delta p^2 - 16\sigma\pi\Delta\mu U})/4\pi\sigma U. \tag{2.8}$$

It follows from (2.5) that the first stationary state is unstable to small perturbations of the radius of the front, while the second is stable, and $R'_{1,2} \sim \exp \omega_{1,2}t$, where

$$\omega_{1,2} = \frac{U}{R^0_{1,2}} \frac{\Delta\mu - \pi\sigma\left(R^0_{1,2}\right)^2}{\Delta\mu + \pi\sigma\left(R^0_{1,2}\right)^2}. \tag{2.9}$$

The critical condition for the vanishing of stationary states is $U = u_m$ (where the points S_1 and S_2 collapse into the point S_{12} of Fig. 1).

When the characteristics of the pump supplying the reacting material are arbitrary, u(R) has the same qualitative form as (2.7), and all conclusions on the number and stability of the stationary states remain in force [15].

3. We consider a small hydrodynamic perturbation of arbitrary norm:

$$\mathbf{V} = \mathbf{V}^0 + \mathbf{V}',\ p = p^0 + p',\ R = R^0 + R', \tag{3.1}$$

where the superscript 0 denotes the stationary values of the quantities and the prime denotes the perturbation.

Equations (1.1), (1.2), and the boundary conditions (1.3)-(1.6) are linear, therefore substitution of (3.1) into these equations leads to equations for the perturbations having the same form as the original equations. In particular, (1.3) and (1.4) take the form

$$\left(p'_{10} - p'_1\right)|_{R_1} = \sigma_1 u'_1|_{R_1}; \tag{3.2}$$

$$\left(p'_2 - p'_{20}\right)|_{R_2} = \sigma_2 u'_2|_{R_2}. \tag{3.3}$$

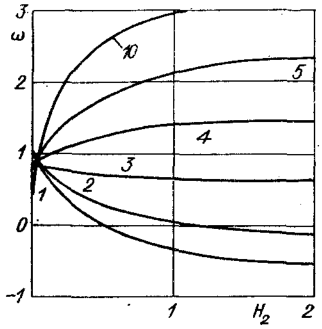

Fig. 5

We estimate the quantities p'_{10} and p'_{20} in the region outside the outer boundary surface (as in the region between the surfaces) we have the Stokes equation $\nabla p'_{20} = \mu_2 \nabla^2 \mathbf{V}'_{20}$ for the perturbations. From this equation and the continuity of the velocity vector on the permeable surfaces we find $p'_{20} \sim \mu_2 u'_{20}(R_2)/\Delta_2$ (Δ_2 is the linear scale of the external region). Comparing this estimate with (3.3), we obtain the relation

$$p'_{20}(R_2)/(p'_2(R_2) - p'_{20}(R_2)) \sim \mu_2/\sigma_2\Delta_2 = H_2, \tag{3.4}$$

showing that penetration of the pressure perturbation into the external region is determined by the ratio of the quantity μ_2/Δ_2 (which can be considered to be the resistance of the external region) to the resistance of the boundary surface σ_2. The parameter H_2 represents the hydraulic analog of the Biot number, which, as is well-known, is the ratio of the thermal resistance of a region Δ/λ to the thermal resistance of its boundary $1/\alpha$ (λ is the thermal conductivity of the medium and α is the coefficient of heat transfer).

In general, the flow in the external region always affects the flow in the interior, and, in particular, it distorts the stationary states considered in Sec. 2 (an exception is the case where the external region is unbounded and the flow is radial and axisymmetric in the external region). Estimates analogous to (3.4) for the stationary states show that these distortions are also determined by the parameter H_2 in practice, the case of small H_2 is of interest, when the distortions of the stationary states are small (according to (3.4), the pressure perturbation in this case is localized in the interior). Then (3.4) can be written in the form $p'_2(R_2) = [1 + O(H_2)]\sigma_2 u'_2(R_2)$, which shows that the penetration of the pressure perturbation into the external region corresponds qualitatively to an increase in the resistance of the boundary surface by a quantity of order H_2. In the zero-order approximation in H_2, we have

$$p'_2(R_2) = \sigma_2 u'_2(R_2). \tag{3.5}$$

An analogous discussion leads to the following result for the inner boundary surface

$$p'_1(R_1) = -\sigma_1 u'_1(R_1). \tag{3.6}$$

It will be shown below that an increase of the resistance of the boundary surface can only stabilize the front, and therefore if the front is stable on the basis of (3.5) and (3.6), it will be even more stable with the use of (3.2) and (3.3).

We substitute (3.1) into the conditions (1.7)-(1.9) on the front and linearize the resulting equation in small perturbations. In linearizing the equation we take into account the axial symmetry of the unperturbed flow, and also the relation

$$F|_R = (F^0 + F')|_{R^0+R'} = F^0|_{R^0} + F'|_{R^0} + \nabla F^0|_{R^0}\cdot\mathbf{n}' + \ldots, \tag{3.7}$$

where F is an arbitrary hydrodynamical quantity and $\mathbf{n}'$ is a small vector determining the perturbation the surface of the front.

After some calculation

$$\mathbf{V}'_1|_{R^0} = \mathbf{V}'_2|_{R^0}; \tag{3.8}$$

$$\mathbf{P}'_1|_{R^0}\cdot\mathbf{n}^0 + \mathbf{P}^0_1|_{R^0}\cdot\nabla R' - 4\mu_1 U R'\mathbf{n}^0/(R^0)^2 = \mathbf{P}'_2|_{R^0}\cdot\mathbf{n}^0 + \mathbf{P}^0_2|_{R^0}\cdot\nabla R' - 4\mu_2 U R'\mathbf{n}^0/(R^0)^2; \tag{3.9}$$

$$\partial R'/\partial t + \mathbf{V}'|_{R^0}\cdot\mathbf{n}^0 + UR'/R^0 = 0. \tag{3.10}$$

We now limit ourselves to the case of angular perturbations ($w_1' = w_2' = 0,\ \partial/\partial z = 0$) and transform to dimensionless variables, expressing times, lengths velocities, and pressures in terms of R^0/U, R^0, U, and $\mu U/R^0$, respectively (the dimensionless variables are denoted by the same symbols as their dimensional analogs). We look for the solution of the linearized problem in the form

$$(u_1', v_1', p_1', u_2', v_2', p_2', R') = (A_1, B_1, e^{-x}C_1/\varepsilon,\ A_2, B_2, e^{-x}C_2, D) \quad \exp(-\omega t + ik\varphi). \tag{3.11}$$

Here A_1, B_1, C_1, A_2, B_2, and C_2 are functions of the variable $x = \ln r$; $\epsilon = \mu_1/\mu_2$; ω is the increment of the perturbations, k is the wave number (an integer), and i is the imaginary unit. An arbitrary angular perturbation can be expanded in a series of harmonics given by (3.11).

Then the equations for the perturbations reduce to a linear system of differential equations with constant coefficients

$$A_{1,2}'' - (1 + k^2) A_{1,2} - 2ikB_{1,2} - C_{1,2}' - C_{1,2} = 0; \tag{3.12}$$

$$2ikA_{1,2} + B_{1,2}'' - (1 + k^2) B_{1,2} - ikC_{1,2} = 0; \tag{3.13}$$

$$A_{1,2}' + A_{1,2} + ikB_{1,2} = 0, \tag{3.14}$$

and the boundary conditions are

$$H_{1,2}C_{1,2}(x_{1,2}) = \mp A_{1,2}(x_{1,2}); \tag{3.15}$$

$$B_{1,2}(x_{1,2}) = 0; \tag{3.16}$$

$$A_1(0) = A_2(0); \tag{3.17}$$

$$B_1(0) = B_2(0); \tag{3.18}$$

$$\varepsilon\left[C_1(0) - 2A_1'(0) - 4D\right] = C_2(0) - 2A_2'(0) - 4D; \tag{3.19}$$

$$\varepsilon\left[B_1(0) - B_1'(0) - ikA_1(0) + 4ikD\right] = B_2(0) - B_2'(0) - ikA_2(0) + 4ikD; \tag{3.20}$$

$$(\omega + 1)D - A_1(0) = 0 \tag{3.21}$$

(a prime denotes differentiation with respect to x).

Integrating the system (3.12)-(3.14), we obtain

$$A_{1,2} = a_{1,2}e^{\lambda_1 x} + b_{1,2}e^{\lambda_2 x} + c_{1,2}e^{\lambda_3 x} + d_{1,2}e^{\lambda_4 x}; \tag{3.22}$$

$$kB_{1,2} = -i\left[a_{1,2}ke^{\lambda_1 x} + b_{1,2}(k-2)e^{\lambda_2 x} - c_{1,2}ke^{\lambda_3 x}\ d_{1,2}(k+2)e^{\lambda_4 x}\right]; \tag{3.23}$$

$$C_{1,2} = 4\left[b_{1,2}(k-1)e^{\lambda_2 x} + d_{1,2}(k+1)e^{\lambda_4 x}\right] \tag{3.24}$$

($\lambda_1 = -\lambda_4 = -(k+1)$, $\lambda_2 = -\lambda_3 = 1 - k$ are the roots of the characteristic equation of the system). When k = 1 (first harmonic) $\lambda_2 = \lambda_3 = 0$ is a multiple root and the solution of the system is

$$A_{1,2} = a_{1,2}e^{2x} + b_{1,2}e^{-2x} + c_{1,2}x + d_{1,2}; \tag{3.25}$$

$$B_{1,2} = i[3a_{1,2}e^{2x} - b_{1,2}e^{-2x} + c_{1,2}(x+1) + d_{1,2}], \tag{3.26}$$

$$C_{1,2} = 8a_{1,2}e^{2x} - 2c_{1,2}. \tag{3.27}$$

Substituting the solution (3.22)-(3.27) into the boundary conditions (3.15)-(3.21), we have a system of nine linear homogeneous equations for the coefficients a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, and D. The condition for the existence of a nontrivial solution of the system is that the determinant be equal to zero:

$$\Delta(F_{mn}) = 0, \tag{3.28}$$

where F_{mn} is the element of the determinant in the m-th row and n-th column (m = 1-9, n = 1-9):

$$F_{11}=F_{21}=R_1^{-k-1},\quad F_{12}=R_1^{-k+1}[1+4H_1(k-1)/k],$$
$$F_{13}=-F_{23}=R_1^{k-1},$$
$$F_{14}=R_1^{k+1}[1+4H_1(k+1)/k],\quad F_{22}=R_1^{-k+1}(k-2)/k,$$
$$F_{24}=-R_1^{k+1}(k+2)/k,$$
$$F_{35}=F_{45}=R_2^{-k-1},\quad F_{36}=R_2^{-k+1}[1-4H_2(k-1)/k],\quad F_{37}=-F_{47}=R_2^{k-1},$$
$$F_{38}=R_2^{k+1}[1-4H_2(k+1)/k],\quad F_{46}=R_2^{-k+1}(k-2)/k,$$
$$F_{48}=-R_2^{k+1}(k+2)/k,$$
$$F_{51}=F_{52}=F_{54}=F_{53}=F_{61}=F_{67}=F_{77}=F_{78}=F_{95}=$$
$$=F_{96}=F_{97}=F_{98}=F_{99}=1,$$
$$F_{55}=F_{56}=F_{57}=F_{58}=F_{63}=F_{65}=F_{75}=F_{76}=-1,$$
$$F_{71}=F_{72}=\varepsilon,\ F_{73}=F_{74}=-\varepsilon,$$
$$F_{81}=F_{84}=\varepsilon(k+1)/k,\ F_{82}=F_{83}=\varepsilon(k-1)/k,$$
$$F_{85}=F_{88}=-(k+1)/k,\ F_{86}=F_{87}=-(k-1)/k,$$
$$F_{89}=2(1-\varepsilon),\ F_{99}=-(\omega+1)$$

(the other elements of the determinant are zero). When k = 1 the elements of the determinant are $F_{12}=\ln R_1-2H_1$, $F_{22}=-(\ln R_1+1)$, $F_{36}=\ln R_2+2H_2$, $F_{46}=-(\ln R_2+1)$, $F_{52}=F_{56}=F_{71}=F_{73}=F_{74}=F_{75}=F_{77}=F_{78}=F_{96}=0$, $F_{72}=-\varepsilon$, $F_{76}=1$.

4. Equation (3.28) gives the dependence of the increment ω on the six dimensionless parameters: R_1, R_2, H_1, H_2, ϵ, k. Expanding the determinant (3.28) by rows containing terms of order ϵ, we obtain

$$\omega(\varepsilon)=(P_0+P_1\varepsilon+P_2\varepsilon^2)/(Q_0+Q_1\varepsilon+Q_2\varepsilon^2). \tag{4.1}$$

Here the coefficients P_1, P_2, Q_1, Q_2 depend on the parameters R_1, R_2, H_1, H_2, k, while the coefficients P_0 and Q_0 depend only on R_2, H_2, k:

$$P_0=[1+2(k+1)H_2]R_2^{2k}-[k^2-4(k^2-2)H_2]R_2^2+$$
$$+2(1+k^2)(1-2H_2)-k^2R_2^{-2}+[1-2(k-1)H_2]R_2^{-2k}; \tag{4.2}$$
$$Q_0=[1+2(k+1)H_2]R_2^{2k}+[k^2-4(k^2-2)H_2]R_2^2+$$
$$+2(1-k^2)(1-2H_2)+k^2R_2^{-2}+[1-2(k-1)H_2]R_2^{-2k}. \tag{4.3}$$

In Fig. 2, in the plane of the parameters R_2 and H_2^{-1}, we have plotted solid and dashed curves for which P_0 and Q_0 are zero, respectively (the numbers next to the curves denote the number of the harmonic). It is evident that in the physical region of the parameters ($R_2 > 1$, $H_2 > 0$) Q_0 is never equal to zero and hence the small parameter ϵ appears in (1.1) as a regular perturbation. In the zero-order approximation in ϵ the increment ω depends only on the parameters with subscript 2, and therefore the stability of the front is determined, to within terms of order ϵ, by the characteristics of the flow behind the front.

The regularity of the solution with respect to ϵ is due to the fact that in the linear approximation in small perturbations the difference between the velocities of the flow and the front is compensated by the change in the distance of the front from the source, and there is no turning of the front (in the linearized Michelson law (3.10) the derivatives $\partial R'/\partial\varphi$, $\partial R'/\partial z$) do not occur). In this approximation the front is oriented normally to the streamlines, and the velocity perturbations and their gradients are of order unity in the entire flow region (in dimensionless variables). The condition that the stresses be equal on the front implies that the stress perturbations on the front are of order ϵ.

It is well known that for channel flow in the presence of a reaction front, accompanied by a strong increase in the viscosity, the small parameter ϵ appears in the solution as a singular perturbation [10]. In this case the difference between the flow and front velocities is "cancelled" by the turning of the front by an angle such that the components of the two velocities along the normal to the front are equal. The front orients itself at a small angle to the streamlines and the velocity and its gradient are of order unity only in the flow region behind the front. It is significant that the stress on the front is also of order unity in this case. From the condition that the stresses on the front be equal it follows that the velocity gradient in the region before the front is of order $1/\epsilon$, which leads to a singularity.

The dependence of the increment on the different parameters was studied numerically. Figure 3 shows the dependence $\omega(\epsilon)$ for $R_1 = 0.28$. $R_2 = 1.5$, $H_1 = H_2 = 0.01$ (the numbers near the curves denote the number of the harmonic k). It is evident that $\omega > 0$ for all harmonics for any value of ϵ and the front is stable. As the viscosity of the product decreases (ϵ increases) the rate of damping of the perturbations increases. When the viscosities of the product and initial mixture are equal ($\epsilon = 1$) perturbations of all wavelengths damp out at the same rate (there is no dispersion). This is because perturbations of the shape of the front do not lead to perturbation of the hydrodynamic fields.

In the case of polymerization the viscosity of the product is 4-6 orders of magnitude larger than the viscosity of the original mixture, and therefore we can limit ourselves to the zero-order approximation in ϵ in (4.1). In this approximation the solid curves in Fig. 2 correspond to $\omega = 0$, while the dashed curves correspond to $\omega^{-1} = 0$ and divide the plane of the parameters into regions constant in sign. For a given value of k, the coefficient $P_0 > 0$ in the region lying above the k-th solid curve, while $Q_0 > 0$ in the region below the k-th dashed curve. The region of instability of the k-th harmonic ($\omega < 0$) is therefore confined between the corresponding solid and dashed curves. It is evident that in the physical region of the parameters the front is unstable in a strip bounded by the straight lines $H_2^{-1} = 0$, $H_2^{-1} = 2$. Here the first harmonic is unstable. The higher harmonics lose stability in small parts of the band bounded above by the corresponding solid curves. These parts of the band correspond to the parameter R_2.

Being close to unity, and therefore the front is close to the outer boundary surface. Hence the front loses stability only when the resistance of the outer boundary surface is small. This result has a simple physical interpretation. Neglecting inertial effects, the stability of the front in the linear approximation in small perturbations is determined by the ratio of the local flow velocity at the location of the perturbed front to its intrinsic propagation velocity. When the front is displaced from the stationary position in the direction away from the axis, the flow velocity at the new position has a tendency to decrease on account of, the increase in the distance from the source, on the one hand, but has a tendency to increase on account of the decrease in the effective resistance of the front due to the increase of its radius, on the other (see (2.2)). If the resistance of the outer boundary surface is large in comparison with the resistance of the front ($H_2 \ll 1$) then the second tendency is suppressed and the front is stable. As the resistance decreases the second tendency begins to dominate and the front loses stability (analogous conclusions can be made for the case when the front is displaced toward the axis).

An example of the dependence of the increment ω on R_2 or different values of k (the numbers labeling the curves) in the instability region of the front ($H_2 = 1$) is given in Fig. 4a, and in the region of absolute stability ($H_2 = 10^{-4}$) in Fig. 4b. The values of R_2 for which ω goes to zero (a) are equal to the abscissas of the points of intersection of the solid curves with the line H_2^{-1} in Fig. 2.

The dependence of ω on H_2 for $R_2 = 1.5$ and different values of k is shown in Fig. 5. It is evident that for harmonics which can lose stability (k = 1, 2) ω decreases with increasing H_2 and therefore an increase of the resistance of the surface (decrease of H_2) can only lead to stabilization of the front.

For small R_1 and large R_2 the boundary surfaces have only a weak effect on the flow near the front and in the limit $R_1 \to 0$, $R_2 \to \infty$ (4.1) reduces to the trivial relation $\omega = -1$. In this case the increment ω ceases to depend on the parameters R_1, R_2, H_1, and H_2, characterizing the properties of the boundary surfaces, and also on ϵ and k, i.e. perturbations of all wavelengths damp out with the same rate. This result shows that in the case of a cylindrical front dispersion of the perturbations is caused by the boundary surfaces.

We note that when $k \neq 0$ a harmonic perturbation does not change the total flux of reacting material $\left(\int_0^{2\pi} e^{ik\varphi}d\varphi = 0\right)$, and therefore the stability of the front to the perturbation does not depend on how material is supplied to the system. On the other hand, when k = 0 (one-dimensional perturbation) the stability of the front is determined by the nature of the supply of material to the system (see Sec. 2). In particular, the case where the reacting material is supplied such that the pressure drop is constant corresponds formally to the use of (3.5) and (3.6) (in which pressure perturbations in the external region are neglected), therefore (2.9) in dimensionless form coincides with (4.1) with $k = 0$.

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